Gravitational waves from binary systems in circular orbits: Convergence of a dressed multipole expansion

Stephen W. Leonard and Eric Poisson
Department of Physics, University of Guelph, Guelph, Ontario N1G 2W1, Canada
(Submitted to Classical and Quantum Gravity, August 17, 1997)

Abstract

The gravitational radiation originating from a compact binary system in circular orbit is usually expressed as an infinite sum over radiative multipole moments. In a slow-motion approximation, each multipole moment is then expressed as a post-Newtonian expansion in powers of v/c, the ratio of the orbital velocity to the speed of light. The "bare multipole truncation" of the radiation consists in keeping only the leading-order (Newtonian) term in the post-Newtonian expansion of each moment, but summing over *all* the multipole moments. In the case of binary systems with small mass ratios, the bare multipole series was shown in a previous paper [Simone *et al.* 1997, *Class. Quantum Grav.* **14**, 237] to converge for all values $v/c < 2/e$, where e is the base of natural logarithms. (These include all physically relevant values for circular inspiral.) In this paper, we extend the analysis to a "dressed multipole truncation" of the radiation, in which the leading-order moments are corrected with terms of relative order $(v/c)^2$ (first post-Newtonian, or 1PN, terms) and $(v/c)^3$ (1.5PN terms). We find that the dressed multipole series converges also for all values $v/c < 2/e$, and that it coincides (within 1 %) with the numerically "exact" results for $v/c < 0.2$. Although the dressed multipole series converges, it is an unphysical approximation, and the issue of the convergence of the true post-Newtonian series remains uncertain. However, our analysis shows that an eventual failure of the true post-Newtonian series to converge cannot originate from summing over the Newtonian, 1PN, and 1.5PN part of all the multipole moments.

Pacs numbers: 0430, 0425N

I. INTRODUCTION

Measurement of gravitational waves from inspiraling compact binaries (composed of neutron stars and/or black holes) by forthcoming laser interferometric detectors (such as LIGO [1] and VIRGO [2]) will rely heavily on theoretical templates obtained from approximations to the exact laws of general relativity [3,4]. One possible method to calculate these templates, which has been pushed to a high degree of accuracy [5–7], is the post-Newtonian approximation, which is based upon the assumption that the orbital motion is slow. Another approach is black-hole perturbation theory [8,9], which is accurate only in the unrealistic situation of systems with small mass ratios. Currently, the post-Newtonian approach appears to be the most promising for generating the templates. Unfortunately, however, there is considerable evidence that the post-Newtonian approximation has poor convergence [10,11], if it converges at all. It is important to understand the reasons for the poor convergence. Although this issue is still an open one, some preliminary steps were taken in this direction in a previous paper [12].

The amount of energy radiated per unit time (denoted $\dot{E}$) by a binary system in circular orbit can be expressed as an infinite sum over the multipole moments of the gravitational-wave field, which are related to the behaviour of the source. In a slow-motion approximation, each multipole moment is then expressed as a post-Newtonian expansion in powers of v/c, the ratio of the orbital velocity to the speed of light. Since higher-order moments come with a higher power of the orbital velocity, the post-Newtonian approximation to $\dot{E}$ consists of truncating the multipole sum at a given multipole order, and in calculating the contributing moments to the appropriate post-Newtonian order. We will refer to this expansion of $\dot{E}$ in powers of v/c as the "true post-Newtonian series". At present, only a few terms of this post-Newtonian series have been calculated [5–7], and the issue of its converge is not at all clear. However, since post-Newtonian corrections can be computed for *all* the multipole moments, the simpler issue of the convergence of the *multipole* sum can be addressed.

In reference [12], the convergence of the multipole series was studied in the restricted context of a "bare multipole truncation", in which the multipole moments were calculated only to leading-order in a post-Newtonian expansion. We refer to these bare moments as "Newtonian", and the bare multipole truncation therefore consists of discarding all post-Newtonian corrections to each of the multipole moments. When an approximation to $\dot{E}$ is constructed by summing over all these bare moments, it is found that the series converges for all orbital velocities less than a critical value. This value depends on the mass ratio of the system; in the case of very small mass ratios, to which we specialize in this paper, the condition for convergence is $v/c < 2/e \simeq 0.7358$, where e is the base of natural logarithms. The convergent interval includes all physically relevant values for the orbital velocity, which must be smaller than its value at the innermost stable circular orbit, $v_{\rm isco}/c = 1/\sqrt{6} \simeq 0.4082$. Although the bare multipole truncation is an unphysical approximation which compares poorly with the numerically "exact" results [10], this analysis reveals that an eventual failure of the true post-Newtonian series to converge cannot originate from summing over the Newtonian part of all the multipole moments.

In this paper, we extend the results of reference [12] to a "dressed multipole truncation", in which the leading-order moments are corrected with terms of relative order $(v/c)^2$ (first post-Newtonian, or 1PN, terms) and $(v/c)^3$ (1.5PN terms). This improved analysis relies

on a previous paper of ours [9], in which the equations of black-hole perturbation theory are integrated for an arbitrary perturbing stress-energy tensor $T^{\alpha\beta}$ (the unperturbed spacetime is assumed to be Schwarzschild). Applying the results of this paper to the particular case of a particle moving on a circular orbit returns the desired post-Newtonian corrections to the bare multipole moments derived in reference [13].

The dressed multipole truncation gives three separate series for $\dot{E}$. The first is the bare multipole series, in which we sum over the Newtonian part of all the multipole moments. The second and third series incorporate the 1PN and 1.5PN corrections to the bare moments, respectively. We find that each of the series converges for $v/c < 2/e$, and we conclude that inclusion of these post-Newtonian corrections does not affect the convergence of the multipole sum. This shows that an eventual failure of the true post-Newtonian series to converge cannot originate from summing over the Newtonian, 1PN, and 1.5PN part of all the multipole moments.

The following sections of the paper provide the details of the calculation. The dressed multipole truncation is introduced in Sec. II, and the convergence of the resulting series is established in Sec. III. Finally, concluding remarks are provided in Sec. IV.

II. DRESSED MULTIPOLE TRUNCATION FOR GRAVITATIONAL RADIATION

In this section, we extend the bare multipole calculation of reference [12] by calculating the 1PN and 1.5PN corrections to the multipole moments of the gravitational-wave field. Such corrections have been calculated for general sources, first by Blanchet [14] using post-Newtonian theory, and then by us [9] using black-hole perturbation theory. To carry out this calculation, we apply the results of reference [9] (hereafter referred to as LP) to the specific case of a point particle moving on a circular orbit around a Schwarzschild black hole. We denote by μ the mass of the particle, and by M the mass of the black hole. It is assumed that $\mu/M \ll 1$. We also denote the orbital radius by r_0, and the angular velocity is $\Omega = (M/{r_0}^3)^{1/2}$. Finally, we define the orbital velocity by $v = \Omega r_0 = (M/r_0)^{1/2}$. Throughout we use units such that $G = c = 1$.

The gravitational-wave luminosity is given by [15]

$$\dot{E} = \frac{1}{32\pi} \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \Big[|\dot{\mathcal{I}}_{lm}(u)|^2 + |\dot{\mathcal{S}}_{lm}(u)|^2 \Big], \tag{1}$$

where $\mathcal{I}_{lm}(u)$ and $\mathcal{S}_{lm}(u)$ are the mass and current multipole moments, respectively, while u denotes retarded time. (Because the orbit is fixed, $\dot{E}$ does not actually vary with u.) To evaluate $\dot{E}$, we shall use equations (5.37)–(5.39) of LP, which give the Fourier transform of the moments, $\tilde{\mathcal{I}}_{lm}(\omega)$ and $\tilde{\mathcal{S}}_{lm}(\omega)$. These equations incorporate post-Newtonian corrections of up to order v^3, relative to the leading-order, Newtonian expressions.

To evaluate the multipole moments, we must first compute the source terms, which are constructed from the stress-energy tensor of the orbiting particle [16]. Using equations (5.24)–(5.27) of LP, we find that the only nonzero source terms are $\rho = R\Delta$, ${}_{-1}j = {}_{-1}J\Delta$, ${}_{1}j = {}_{1}J\Delta$, and ${}_{0}t = {}_{0}T\Delta$, where

$$R = 1 + \frac{3v^2}{2} + O(v^4), \qquad {}_{-1}J = \frac{-iv}{\sqrt{2}}\left[1 + \frac{3v^2}{2} + O(v^4)\right],$$

$${}_0T = \frac{v^2}{2}\left[1 + \frac{3v^2}{2} + O(v^4)\right], \qquad \Delta = \frac{\mu}{r_0^2}\,\delta(r - r_0)\delta(\cos\theta)\delta(\phi - \Omega t), \tag{2}$$

and ${}_1J = -\left({}_{-1}J\right)$.

The second step is to substitute these results into equations (5.37)–(5.39) of LP. After a few steps of algebra, the mass and current multipole moments become

$$\begin{aligned} \tilde{\mathcal{I}}_{lm}(\omega) &= \frac{16\pi\mu}{(2l+1)!!}\left[\frac{(l+1)(l+2)}{2(l-1)l}\right]^{1/2} \mathcal{T}_l(\omega)\,(-i\omega r_0)^l\,A_l(\omega) \\ &\quad \times (R + 2{}_0T)\,Y_{lm}(\tfrac{\pi}{2}, 0)\,\delta(\omega - m\Omega), \end{aligned} \tag{3}$$

$$\begin{aligned} \tilde{\mathcal{S}}_{lm}(\omega) &= \frac{-16\pi i\mu}{(2l+1)!!}\left(\frac{l+2}{l-1}\right)^{1/2} \mathcal{T}_l^{\#}(\omega)\,(-i\omega r_0)^l\,B_l(\omega) \\ &\quad \times \left[{}_{-1}J\,{}_{-1}Y_{lm}(\tfrac{\pi}{2}, 0) + {}_1J\,{}_1Y_{lm}(\tfrac{\pi}{2}, 0)\right]\,\delta(\omega - m\Omega), \end{aligned} \tag{4}$$

where

$$\begin{aligned} A_l(\omega) &= 1 - \frac{l+9}{2(l+1)(2l+3)}(\omega r_0)^2 - (l+2)\frac{M}{r} + O(v^4), \\ B_l(\omega) &= 1 - \frac{l+4}{2(l+2)(2l+3)}(\omega r_0)^2 - \frac{(l-1)(l+2)}{l}\frac{M}{r} + O(v^4), \\ |\mathcal{T}_l(\omega)|^2 &= |\mathcal{T}_l^{\#}(\omega)|^2 = 1 + 2\pi M\omega + O(v^6), \end{aligned} \tag{5}$$

while ${}_1Y_{lm}(\theta, \phi)$ and ${}_{-1}Y_{lm}(\theta, \phi)$ are the spherical harmonics of spin weight 1 and -1, respectively [17].

The third step is to invert the Fourier transform, which is trivial because of the delta function in the integrand. This yields

$$\begin{aligned} \mathcal{I}_{lm}(u) &= \frac{16\pi\mu}{(2l+1)!!}\left[\frac{(l+1)(l+2)}{2(l-1)l}\right]^{1/2} \mathcal{T}_l(m\Omega)\,(-im\Omega r_0)^l\,A_l(m\Omega) \\ &\quad \times (R + 2{}_0T)\,Y_{lm}(\tfrac{\pi}{2}, 0)\,e^{-im\Omega u}, \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{S}_{lm}(u) &= \frac{-16\pi i\mu}{(2l+1)!!}\left(\frac{l+2}{l-1}\right)^{1/2} \mathcal{T}_l^{\#}(m\Omega)\,(-im\Omega r_0)^l\,B_l(m\Omega) \\ &\quad \times \left[{}_{-1}J\,{}_{-1}Y_{lm}(\tfrac{\pi}{2}, 0) + {}_1J\,{}_1Y_{lm}(\tfrac{\pi}{2}, 0)\right]\,e^{-im\Omega u}. \end{aligned} \tag{7}$$

It is now useful to write explicit expressions for the spin-weighted spherical harmonics [13]. First, if $l + m$ is even, then

$$Y_{lm}(\tfrac{\pi}{2}, 0) = (-1)^{(l+m)/2}\left(\frac{2l+1}{4\pi}\right)^{1/2}\frac{[(l-m)!(l+m)!]^{1/2}}{(l-m)!!(l+m)!!}, \tag{8}$$

while $Y_{lm}(\frac{\pi}{2}, 0) = 0$ if $l + m$ is odd. Second, if $l + m$ is odd, then

$$
{}_{-1}Y_{lm}(\tfrac{\pi}{2},0) = (-1)^{(l+m)/2}\left[\frac{2l+1}{4\pi l(l+1)}\right]^{1/2}\frac{(l+m)!!(l-m)!!}{[(l+m)!(l-m)!]^{1/2}}; \tag{9}
$$

although ${}_{-1}Y_{lm}(\frac{\pi}{2},0) \neq 0$ for $l+m$ even, its expression will not be needed. Finally, we note that ${}_{1}Y_{lm}(\frac{\pi}{2},0) = (-1)^{l+m}{}_{-1}Y_{lm}(\frac{\pi}{2},0)$.

Looking at this last result and the fact that ${}_{1}J = -({}_{-1}J)$, we see that the quantity within the square brackets of equation (7) may be simplified as follows: it vanishes identically for $l+m$ even, and it is equal to $2{}_{-1}J\,{}_{-1}Y_{lm}(\frac{\pi}{2},0)$ when $l+m$ is odd. Using this while substituting equations (6) and (7) into equation (1), we obtain

$$
\dot{E} = \dot{E}_Q \sum_{l=2}^{\infty}\sum_{m=-l}^{l} \tfrac{1}{2}\eta_{lm} = \dot{E}_Q \sum_{l=2}^{\infty}\sum_{m=1}^{l} \eta_{lm}, \tag{10}
$$

where

$$
\dot{E}_Q = \frac{32}{5}\left(\frac{\mu}{M}\right)^2 v^{10} \tag{11}
$$

is the quadrupole-formula result. We also have

$$
\eta_{lm} = \frac{5\pi}{4}\frac{m^{2(l+1)}}{(2l+1)!!^2} v^{2(l-2)}\left(1+2\pi m v^3\right)\zeta_{lm}, \tag{12}
$$

where

$$
\zeta_{lm} = \frac{(l+1)(l+2)}{2(l-1)l}\left[A_l(R+2_0T)\,Y_{lm}(\tfrac{\pi}{2},0)\right]^2 \tag{13}
$$

if $l+m$ is even, while

$$
\zeta_{lm} = \frac{l+2}{l-1}\left[2B_l\,{}_{-1}J\,{}_{-1}Y_{lm}(\tfrac{\pi}{2},0)\right]^2 \tag{14}
$$

if $l+m$ is odd. The symmetry $\eta_{l,-m} = \eta_{lm}$, used in equation (10), can easily be established from the previous expressions.

Equations (10)–(14) constitute the dressed multipole truncation of the gravitational-wave luminosity. The right-hand side of equation (10) is an infinite series in powers of v, and our next task is to examine the convergence of this series.

III. CONVERGENCE OF DRESSED MULTIPOLE TRUNCATION

The velocity dependence of all the terms in equation (10) is made clear by substituting equations (2) and (5), which gives

$$
\dot{E} = \dot{E}_Q \sum_{l=2}^{\infty} v^{2(l-2)} \sum_{m=1}^{l} \begin{cases} (a_{lm} + b_{lm}v^2 + c_{lm}v^3) & l+m \text{ even} \\ v^2\,(d_{lm} + e_{lm}v^2 + f_{lm}v^3) & l+m \text{ odd} \end{cases}. \tag{15}
$$

The various coefficients are

$$a_{lm} = \frac{5\pi}{4}\frac{m^{2(l+1)}}{(2l+1)!!^2}\frac{(l+1)(l+2)}{(l-1)l}\left[Y_{lm}(\tfrac{\pi}{2},0)\right]^2, \tag{16}$$

$$b_{lm} = -\left[\frac{(l+9)m^2}{(l+1)(2l+3)} + 2l - 1\right] a_{lm}, \tag{17}$$

$$c_{lm} = 2\pi m\, a_{lm}, \tag{18}$$

$$d_{lm} = 5\pi\, \frac{m^{2(l+1)}}{(2l+1)!!^2}\frac{l+2}{l-1}\left[{}_{-1}Y_{lm}(\tfrac{\pi}{2},0)\right]^2, \tag{19}$$

$$e_{lm} = -\left[\frac{(l+4)m^2}{(l+2)(2l+3)} + \frac{2(l-1)(l+2)}{l} - 3\right] d_{lm}, \tag{20}$$

$$f_{lm} = 2\pi m\, d_{lm}. \tag{21}$$

We now express equation (15) in the following form, which displays a more transparent grouping of terms:

$$\dot{E} = \dot{E}_Q\left\{a_{22} + b_{22}v^2 + c_{22}v^3 + \sum_{l=3}^{\infty} v^{2(l-2)}\Big[S_1(v) + v^2S_2(v) + v^3S_3(v)\Big]\right\}, \tag{22}$$

where

$$S_1(v) = \sum_{l=3}^{\infty} v^{2(l-2)}a_{ll}\left[1 + \frac{1}{a_{ll}}\sum_{m=1}^{l-1}(a_{lm} + d_{l-1,m})\right], \tag{23}$$

$$S_2(v) = \sum_{l=3}^{\infty} v^{2(l-2)}b_{ll}\left[1 + \frac{1}{b_{ll}}\sum_{m=1}^{l-1}(b_{lm} + e_{l-1,m})\right], \tag{24}$$

$$S_3(v) = \sum_{l=3}^{\infty} v^{2(l-2)}c_{ll}\left[1 + \frac{1}{c_{ll}}\sum_{m=1}^{l-1}(c_{lm} + f_{l-1,m})\right]. \tag{25}$$

The dressed multipole truncation therefore splits $\dot{E}$ into three separate series: $S_1(v)$, the bare multipole series, $v^2S_2(v)$, the 1PN correction, and $v^3S_3(v)$, the 1.5PN correction. It is important to point out that in equations (23)–(25), the sums over m are restricted to values such that $l+m$ is *even*.

The convergence of the dressed multipole series may be established by analyzing the convergence of each of the partial series. We begin with $S_1(v)$, the bare multipole series of reference [12]. Its convergence is established in the following manner. First, it is shown (numerically) that the terms in the square brackets of equation (23) approach the approximate value 1.01 as $l \to \infty$. This implies that the behaviour of the series for large l is dominated by the factor in front of the square brackets. The series may therefore be approximated by $S_1(v) \simeq \sum_{l=3}^{\infty} v^{2(l-2)}a_{ll}$, which is shown (using the Cauchy ratio test) to converge for

$$v < 2/e \simeq 0.7358. \tag{26}$$

The convergence of $S_2(v)$ is established in the same manner. First, one shows numerically that the terms in the square brackets of equation (24) also approach 1.01 as $l \to \infty$. The series is then approximated by $S_2(v) \simeq \sum_{l=3}^{\infty} v^{2(l-2)}b_{ll}$, and the convergence of this series is again analyzed via the Cauchy ratio test. Using equation (17), we find that $b_{l+1,l+1}/b_{ll}$

is equal to $a_{l+1,l+1}/a_{ll}$ multiplied by a ratio of terms quadratic in l. In the limit $l \to \infty$, $b_{l+1,l+1}/b_{ll} \sim a_{l+1,l+1}/a_{ll}$ and we immediately conclude that the convergence of $S_2(v)$ is also determined by equation (26).

The same is true also for $S_3(v)$. Again, it can be shown that the terms in the square brackets of equation (25) approach 1.01 as $l \to \infty$. The series is then approximated by $S_3(v) \simeq \sum_{l=3}^{\infty} v^{2(l-2)} c_{ll}$. From equation (18) we see that in the limit $l \to \infty$, the ratio of successive coefficients is equal to $a_{l+1,l+1}/a_{ll}$. This allows us to conclude that the convergence of $S_3(v)$ is also determined by equation (26).

In summary, when 1PN and 1.5PN corrections are added to the bare multipole series, $\dot{E}$ splits naturally into three separate series, which all converge for $v < 2/e$. Therefore, the dressed multipole truncation of $\dot{E}$ must also converge in this interval. We note that this convergence test ignores the fact that the combined series $S(v) \equiv S_1(v) + v^2 S_2(v) + v^3 S_3(v)$ is alternating at large l. However, establishing the convergence of the alternating series using the Leibnitz criterion does not result in an improvement on equation (26). Furthermore, a numerical analysis of the convergence of $S(v)$ confirms equation (26).

IV. CONCLUDING REMARKS

We have shown that for circular binaries with small mass ratios, the dressed multipole truncation leads to a convergent series for the gravitational-wave luminosity, for all physically relevant values of the orbital velocity. It appears likely that this conclusion will remain valid for binaries of comparable masses, but a separate calculation is required to confirm this. Our results imply that the poor convergence of the true post-Newtonian series does not originate from summing over the Newtonian, 1PN, and 1.5PN part of all the multipole moments.

We see in figure 1 that the dressed-multipole series leads to a significant improvement in accuracy, compared with the bare-multipole series. By including the negative 1PN corrections and the (smaller) positive 1.5PN corrections, our approximation to the luminosity coincides (within 1 %) with the numerically "exact" results [10] over the interval $v < 0.2$. One would therefore expect that the incorporation of further corrections would lead to a series that converges everywhere in the region of physical interest, and stays accurate over a larger interval.

Meanwhile, the issue of the convergence of the true post-Newtonian series remains an open one.

ACKNOWLEDGMENTS

This research was supported by the Natural Sciences and Engineering Research Council of Canada. Conversations with Clifford Will were greatly appreciated.

REFERENCES

[1] Abramovici A, Althouse W E, Drever R W P , Gürsel Y, Kawamura S, Raab F J, Shoemaker D, Sievers L, Spero R E, Thorne K S, Vogt R E, Weiss R, Whitcomb S E and Zucker M E 1992 *Science* **256**, 325

[2] Bradaschia C, Del Fabbro R, Di Virgilio A, Giazotto A, Kautzky H, Montelatici V, Passuello D, Brillet A, Cregut O, Hello P, Man C N, Manh P T, Marraud A, Shoemaker D, Linet J Y, Barone F, Di Fiore L, Milano L, Russo G, Aguirregabiria J M, Bel H, Duruisseau J P, Le Denmat G, Tourrenc Ph, Capozzi M, Longo M, Lops M, Pinto I, Rotoli G, Damour T, Bonazzola S, Marck J A, Gourghoulon Y, Holloway L E, Fuligni F, Iafolla V and Natale G 1990 *Nucl. Instrum. & Methods* **A289**, 518

[3] Cutler C and Flanagan E E 1994 *Phys. Rev. D* **49** 2658

[4] Poisson E and Will C M 1995 *Phys. Rev. D* **52** 848

[5] Blanchet L, Damour T, Iyer B R, Will C M and Wiseman A G 1995 *Phys. Rev. Lett.* **74**, 3515

[6] Will C M and Wiseman A G 1996 *Phys. Rev. D* **54**, 4813

[7] Blanchet L 1996 *Phys. Rev. D* **54** 1417

[8] Tanaka T, Tagoshi H and Sasaki M 1996 *Prog. Theor. Phys.* 96, 1087

[9] Leonard S W and Poisson E 1997 *Phys. Rev. D*, in press (gr-qc/9705014)

[10] Poisson E 1995 *Phys. Rev. D* **52**, 5719; Erratum and Addendum 1997 *ibid.* **55**, 7980

[11] Simone L E, Poisson E and Will C M *Phys. Rev. D* **52**, 4481

[12] Simone L E, Leonard S W, Poisson E and Will C M 1997 *Class. Quantum Grav.* **14** 237

[13] Poisson E and Sasaki M 1995 *Phys. Rev. D* **51** 5753

[14] Blanchet L 1995 *Phys. Rev. D* **51** 2559

[15] Thorne K S 1980, *Rev. Mod. Phys.* **52** 299

[16] Poisson E 1993 *Phys. Rev. D* **47** 1497

[17] Goldberg J N, MacFarlane A J, Newman E T, Rohrlich F and Sudarshan E C G 1967 *J. Math. Phys.* **8** 2155

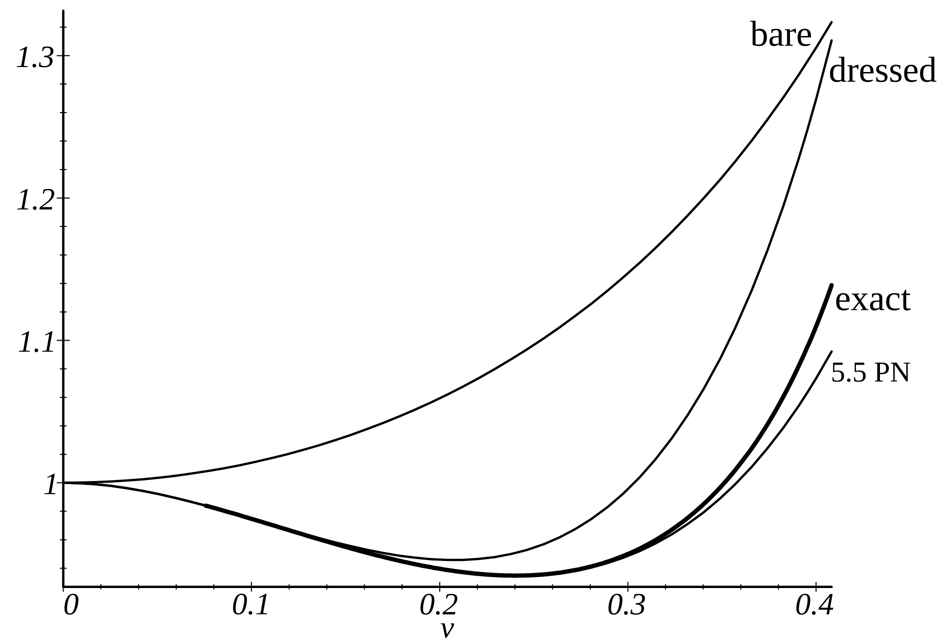

FIG. 1. Plots of $\dot{E}/\dot{E}_Q$ as a function of orbital velocity v. The thick curve is the "exact" curve, obtained by numerically integrating the equations of black-hole perturbation theory (see reference [10]). The curve immediately below the exact curve is the 5.5PN curve, obtained from the post-Newtonian expansion (carried up to order v^{11} beyond the quadrupole formula) of Tanaka *et al.* [8]. The first curve above the exact curve is the dressed multipole truncation, obtained from equation (22). Finally, the highest curve is the bare multipole truncation, obtained also from equation (22), but with $S_2(v)$ and $S_3(v)$ set to zero. We see that the accuracy of the dressed multipole truncation is far superior to that of the bare multipole truncation, but that it does not match the accuracy of the 5.5PN approximation.